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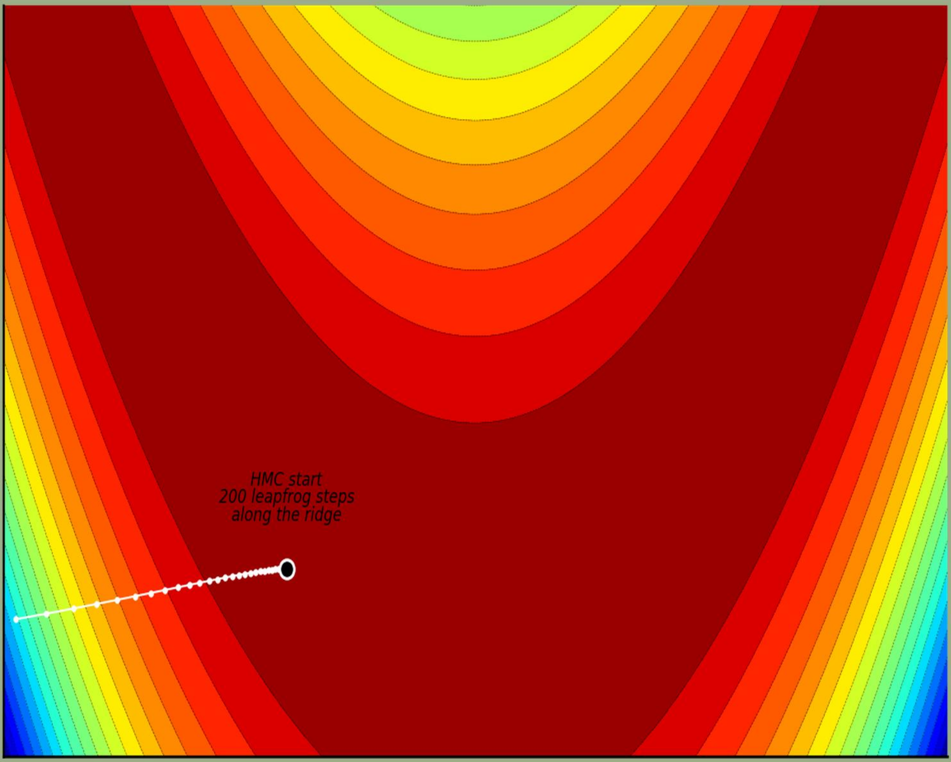

Monte Carlo Methods for Non-Life Actuaries

Reserve Risk, Aggregate Losses, and Capital Allocation in R

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Preface

Non-life actuaries have read Glasserman's "Monte Carlo Methods in Financial Engineering" for two decades and learned variance reduction, low-discrepancy sequences, and the MCSE-based reporting discipline that the book established as the field's professional standard. They have read Mack's 1993 ASTIN paper through the Wüthrich–Merz monograph and absorbed the analytic prediction error for the chain ladder. They have read Klugman, Panjer, and Willmot's "Loss Models" and absorbed the compound-distribution machinery. The three traditions sit on a non-life reserving actuary's desk, and each is excellent on its own ground.

Each is also incomplete for the question a non-life internal-model team is actually asked at year-end. Glasserman is calibrated against derivatives books, not motor TPL triangles. Mack is the analytic core but is silent on the catastrophe overlay, the copula coupling, the reinsurance treaty layer, and the capital allocation. Klugman covers the compound aggregate but stops short of the Solvency II one-year horizon and the regulator-facing audit defense. The actuary who wants to ship a defensible internal-model submission must currently stitch the three traditions together from scratch, inventing the methodology disclosure and the diagnostic suite anew at every annual recalibration.

This book is that stitch, written explicitly for the non-life setting. It covers the full pipeline from per-claim severity to the firm-level economic capital model, with the Solvency II one-year horizon as the binding regulatory framework throughout. Every numerical result in the chapters is produced by R code printed in Appendix A and reproducible by the reader; every methodology choice is named, defended, and routed against the regulator's likely audit response. The audience is the practitioner who already knows variance reduction and MCSE — and who wants the non-life specialisation the existing literature does not provide.

The book delivers five things the existing literature does not assemble together in a single defensible pipeline. First, the reserve-risk machinery on real triangles: bootstrap chain ladder, Mack analytic, GLM Tweedie, and Bayesian hierarchical, with the cross-method reconciliation that makes the twenty-to-forty-percent gap between bootstrap SD and Mack analytic prediction error visible rather than buried. Second, the frequency–severity–tail engine: Poisson and negative-binomial count models, Panjer recursion, peaks-over-threshold with the GPD threshold-selection discipline a regulator will audit, and the spliced and generalized-hyperbolic alternatives when POT fails. Third, the multi-line aggregation: t-copula and R-vine families calibrated against the joint-extreme evidence the calibration window allows, with the EVT-heuristic bound that quantifies what the data cannot anchor. Fourth, the reinsurance and catastrophe machinery: layer pricing under the Mildenhall decomposition with Bühlmann–Straub credibility on thin-experience layers, and the ELT-driven cat module with the secondary-uncertainty discipline that catches the demand-surge double count my Munich Re years taught me to refuse. Fifth, the Solvency II finishing pipeline: Merz–Wüthrich one-year reserve risk, two-state Markov underwriting cycle for premium risk, Euler and Aumann–Shapley capital allocation under coherent risk measures, and the full economic capital model on a four-line plus cat-overlay portfolio.

The chapters proceed in five movements. Chapter 1 fixes the Monte Carlo setting — LLN, CLT, MCSE, RNG discipline — that every subsequent chapter relies on. Chapters 2 through 5 are the reserving toolkit: bootstrap chain ladder, Mack's distribution-free model, GLM stochastic reserving across the Tweedie family, and the Bayesian MCMC interlude with Hamiltonian Monte Carlo and the no-U-turn sampler. Chapters 6 and 7 are the frequency–severity engine: Poisson and negative-binomial count models with Panjer aggregation; peaks-over-threshold severity tails with spliced and generalized-hyperbolic alternatives. Chapter 8 calibrates the t-copula. Chapter 9 prices reinsurance layers. Chapter 10 integrates the vendor catastrophe model. Chapter 11 produces the aggregate portfolio loss distribution with a three-method cross-validation (simulation, FFT, Panjer) that audits the tail. Chapters 12 and 13 deliver the Solvency II reserve-risk and premium-risk modules. Chapter 14 develops capital allocation. Chapter 15 assembles everything into the firm-level economic capital model on the calibrated multiline_casualty and property_continental cat-overlay portfolio.

Appendix A prints the decisive R script for each chapter — fifteen scripts that together reproduce every printed numerical result. Appendix B carries one hundred and five applied exercises in seven exercises per chapter, organised by chapter and covering the methodology choices a non-life actuary faces in the year-end review and the regulatory submission. Both appendices are integral to the book; the chapters develop the machinery, the appendices show the reader how the machinery is run and how it is defended.

The book's defensibility-first orientation is the orientation I have brought to twenty-six years of year-end sign-offs across the Nordic primary and reinsurance markets — eleven at If P&C, twelve at Munich Re Stockholm, and the last three as an independent consultant on Solvency II internal-model recalibrations. Every methodology choice in the chapters is named, every diagnostic is run, and every result that lands in the SCR submission is reproducible from the code in Appendix A. The book does not pretend that Monte Carlo simulation produces capital numbers without methodology commitments; it makes the commitments explicit and shows the reader how to defend them when the Finansinspektionen, BaFin, or the PRA asks.

Sigrid Norrgård. Saltsjöbaden, 2026.

Contents

Chapter 1

The Actuarial Monte Carlo Setting

Every reserve $\widehat{R}$, every capital requirement, every reinsurance price this book reports is the output of a Monte Carlo run, and a Monte Carlo run is only as honest as the discipline the actuary brought to it. Actuarial Monte Carlo is not generic statistical simulation — it is a regulator-facing inferential practice with binding standards on the standard error reported, the random stream that produced it, and the audit trail back to the code that ran. The opening chapter fixes those standards.

The deliverable is a single calibrated baseline for the `motor_nordic` reserve estimator on which the rest of the book reports its measurements. That baseline is the naive Monte Carlo estimator $\hat{\mu}_N$ run under a parallel-safe L'Ecuyer-CMRG generator with documented substreams; the operational signature is a measured Monte Carlo standard error on every quoted figure, with the headline closing artifact being a variance-reduction factor table that benchmarks antithetic, control-variate, stratified, and common-random-numbers reductions against the unreduced baseline. A reserve quoted without its MCSE envelope is, by my reading after twenty-six years of sign-offs, professional malpractice — and the protocol this chapter sets is what makes the refusal mechanically possible.

The chapter is built in five sections. §1.1 develops the naive Monte Carlo estimator, the strong law and central limit theorem that govern it, and the MCSE reporting protocol every later chapter inherits. §1.2 fixes the L'Ecuyer-CMRG pseudo-random stream and the substream-allocation pattern for parallel work. §1.3 introduces antithetic variates with the monotoneintegrand variance bound. §1.4 develops the control-variate and stratified-sampling orthogonal levers. §1.5 closes with the common-random-numbers paired-comparison driver and the variance-reduction factor table the rest of the book cites.

1.1 The Monte Carlo estimator, its SLLN, and the MCSE obligation

The operational discipline of regulator-grade actuarial Monte Carlo rests on five commitments made once and inherited by every later chapter. Every reported number carries the Monte Carlo standard error that produced it; the pseudo-random stream is a parallel-safe L'Ecuyer-CMRG generator with named substreams, never an unqualified seed on a Mersenne Twister default; variance-reduction levers are deployed on a single named baseline simulator with realized variance ratios *measured* against that baseline rather than asserted theoretically; every method-versus-method comparison runs under common random numbers unless the methods explicitly cannot share inputs; every quoted number traces to a named script in `book_code/` with a documented seed. The baseline portfolio is `motor_nordic` — a long-tail Nordic motor third-party-liability book whose calibration constants live in `book_meta.yaml`. Subsequent chapters of this book refine the same baseline under successively more elaborate reserve estimators; the variance-reduction factor table that closes this chapter is the scaffold against which those chapters report MCSE deltas.

Fix a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and a real-valued, integrable random variable $X\colon \Omega \to \mathbb{R}$ with distribution F. Let $g\colon \mathbb{R} \to \mathbb{R}$ be a measurable function with $\mathbb{E}|g(X)| < \infty$. The quantity of interest is

$$\mu \;=\; \mathbb{E}[g(X)] \;=\; \int_{\mathbb{R}} g(x)\, dF(x). \tag{1.1}$$

In the `motor_nordic` setting, $X = (Z_1, \dots, Z_{10})$ is the vector of ten log-increments of the cumulative paid loss across development periods, F is the joint lognormal distribution with the calibrated (μ_0, σ), and $g(X) = \exp\!\big(\sum_{j=1}^{10} Z_j\big)$ is the ultimate loss per accident year. The integral in (**??**) has no closed form for the joint distribution actuaries actually fit, and so a sample mean is what gets reported.

The naive Monte Carlo estimator of μ, based on N independent and identically distributed replicates $X_1, \dots, X_N$ drawn from F, is

$$\hat{\mu}_N = \frac{1}{N}\sum_{b=1}^{N} g(X_b). \tag{1.2}$$

The estimator is unbiased by linearity of expectation: $\mathbb{E}[\hat{\mu}_N] = \mu$ for every finite N, provided μ exists. Two non-trivial results govern the rest of this book: the strong law of large numbers, which says $\hat{\mu}_N$ converges to μ almost surely; and the central limit theorem, which says the residual error $\hat{\mu}_N - \mu$ is approximately Gaussian with standard deviation $\sigma_g/\sqrt{N}$ for all sufficiently large N, where $\sigma_g^2 = \operatorname{Var} g(X)$.

1.1.1 Strong law of large numbers

Theorem 1.1 (SLLN for the Monte Carlo estimator). *Let $X_1, X_2, \ldots$ be i.i.d. with distribution F, and let g be measurable with $\mathbb{E}|g(X_1)| < \infty$. Then*

$$\hat{\mu}_N \xrightarrow{\text{a.s.}} \mu \quad \text{as } N \to \infty. \tag{1.3}$$

Proof. Set $Y_b = g(X_b)$. Then $Y_1, Y_2, \ldots$ are i.i.d. with finite mean $\mu = \mathbb{E}[Y_1]$. Kolmogorov's strong law for i.i.d. integrable sequences [1] states that $N^{-1}\sum_{b=1}^{N} Y_b \to \mathbb{E}[Y_1]$ almost surely. The conclusion follows by identifying the partial sums on the left with $\hat{\mu}_N$. □

Theorem 1.1 is the existence theorem for Monte Carlo. It guarantees that, given enough replicates, the simulator's reported mean converges to the population mean. It says nothing about how *fast* the convergence happens, and it does not on its own quantify the simulator's uncertainty at a finite N. The standard error at finite N is the central limit theorem's contribution.

1.1.2 Central limit theorem and the MCSE bound

Theorem 1.2 (CLT for the Monte Carlo estimator). *Let $X_1, X_2, \ldots$ be i.i.d. with distribution F and $\sigma_g^2 = \operatorname{Var} g(X_1) < \infty$. Then*

$$\sqrt{N}\,(\hat{\mu}_N - \mu) \overset{\text{d}}{\to} \mathcal{N}(0, \sigma_g^2) \quad \text{as } N \to \infty. \tag{1.4}$$

Proof. The summands $Y_b - \mu$ are i.i.d. with mean zero and variance $\sigma_g^2 \in (0, \infty)$. The Lindeberg-Lévy CLT applies. □

The standard Monte Carlo reference for the standard-error envelope and the practical implications of the CLT is [2]. The CLT delivers the **Monte Carlo standard error** of $\hat{\mu}_N$:

Chapter 3

Mack's Distribution-Free Model

Chapter 2 closed with the residual-bootstrap reserve distribution wrapped around the chain-ladder point estimate $\widehat{R}$. That envelope cost a distributional commitment — the over-dispersed-Poisson family on the incremental triangle — and a few thousand replicates of CPU time. For the Solvency II SCR calculation the cost is acceptable; for a back-of-envelope second opinion at year-end committee, or for a reserve-risk number that has to be defended without an exponential-family assumption the data may or may not support, the bootstrap is too much machinery for the question being asked.

This chapter delivers the analytic alternative. Mack's 1993 framework keeps the same chain-ladder point estimate $\hat{f}_j$ but retreats the inferential commitment to two moment assumptions — a conditional mean and a conditional variance parameterized by a single development-period volatility σ_j — and derives the mean square error of prediction of the reserve in closed form, no resampling required. The deliverable is the analytic mean-plus-MSE engine, a parametric-bootstrap variant that converts those moments into a full reserve distribution when the regulator's Q(0.995) reading needs one, a multi-line extension for portfolios where each line is fit independently, and the empirical reconciliation against the Chapter 2 bootstrap that closes the analytic-vs-simulation equivalence. Quoting the bootstrap's standard deviation as Mack's analytic prediction error — they are not the same quantity and differ by twenty to forty percent on a typical motor portfolio — is the kind of conflation I have walked into reserving committees ready to refuse.

The chapter is built in five sections. §3.1 fixes the two moment assumptions and the unbiased age-to-age sigma estimator. §3.2 derives the closed-form

MSEP recursion and the asymptotic-normality result. §3.3 turns the analytic moments into a full reserve distribution via the parametric bootstrap. §3.4 extends to multi-line portfolios with the Quarg-Mack trend correction and the sigma-floor heuristic. §3.5 closes the chapter with the Mack-vs-BCL bootstrap comparison on `motor_nordic`, `property_continental`, and `multiline_casualty`.

3.1 Mack's first- and second-moment assumptions and the age-to-age sigma estimator

Let $C_{i,j}$ denote the cumulative paid loss for accident year $i \in \{1, \dots, I\}$ at development period $j \in \{1, \dots, J\}$, with the upper-left triangle observed and the lower-right triangle awaiting projection. Mack [10] (later extended in [15]) introduced a two-assumption framework that makes the chain-ladder estimator inferentially honest without committing to any distribution.

The framework's central move is the *retreat* from a full likelihood to a moment specification. The Chapter 2 ODP bootstrap treated the chain-ladder point estimate as the MLE under the over-dispersed-Poisson family on the incremental triangle — a distributional commitment that delivered Pearson-residual exchangeability and the full reserve distribution at the cost of a likelihood the data must support. Mack's framework keeps the chain-ladder point estimate but reads it through a *different* inferential lens: the cumulative paid is modeled as a stochastic process whose conditional first moment is the volume-weighted development factor and whose conditional second moment is parameterized by a single per-column volatility σ_j. The third moment, the parametric family, the residual distribution — none of that is specified. The estimator is what it always was; the inferential machinery underneath is leaner.

The reduced commitment buys robustness. A portfolio whose incremental triangle fails the ODP Pearson-residual heteroscedasticity diagnostic of Chapter 2 §2.2 may still satisfy Mack's mean-and-variance specification, and the analytic MSEP delivered by §3.2 is then the right inferential quantity. The cost is that the framework no longer produces a *distribution* — only a mean and a variance. §3.3 closes that gap with a parametric bootstrap whose distributional commitment is taken later, at the reporting stage, rather than embedded in the likelihood. The result is a two-stage discipline — analytic moments first, parametric distribution second — that splits the modeling commitments the bootstrap of Chapter 2 made jointly.

Assumption M1 (Conditional mean). *There exist constants* $f_1, \dots, f_{J-1}$

such that for every accident year i and development period $j \leq J-1$,

$$\mathbb{E}[C_{i,j+1} \,|\, C_{i,1}, \dots, C_{i,j}] \;=\; f_j \cdot C_{i,j}. \tag{3.1}$$

Assumption M2 (Conditional variance). *There exist constants $\sigma_1^2, \dots, \sigma_{J-1}^2 > 0$ such that for every accident year i and development period $j \leq J-1$,*

$$\mathrm{Var}(C_{i,j+1} \,|\, C_{i,1}, \dots, C_{i,j}) \;=\; \sigma_j^2 \cdot C_{i,j}. \tag{3.2}$$

Assumption M3 (Cross-accident-year independence). *Accident years are independent: $C_{i,\cdot}$ and $C_{k,\cdot}$ are independent for $i \neq k$.*

Assumptions M1 and M2 are weaker than the ODP family of Chapter 2: M2 declares only the *shape* of the conditional variance (proportional to the current cumulative paid), without committing to a likelihood. The bootstrap of Chapter 2 needed the ODP family to define Pearson residuals as exchangeable; Mack's model does not need exchangeability. M1 is the *same* deterministic chain-ladder relation as before, now stated as a conditional mean rather than as a defining formula. M3 — accident-year independence — was implicit in the bootstrap-resampling step of Algorithm 2.8 and is now declared explicitly.

Definition 3.1 (Development factor under Mack's model). *The volume-weighted estimator of the development factor f_j from the observed upper-left triangle is*

$$\hat{f}_j \;=\; \frac{\sum_{i=1}^{I-j} C_{i,j+1}}{\sum_{i=1}^{I-j} C_{i,j}}, \qquad j = 1, \dots, J-1. \tag{3.3}$$

The estimator $\hat{f}_j$ is identical to the volume-weighted chain-ladder factor of Definition 2.1. What changes is the inferential interpretation: under M1, $\hat{f}_j$ is the unique best linear unbiased estimator (BLUE) of f_j given the observed column-j and column-$j+1$ cells, and its conditional variance is what M2 will deliver.

Definition 3.2 (Age-to-age sigma estimator). *The Mack 1993 estimator of the development-period volatility σ_j^2 under M2 is*

$$\hat{\sigma}_j^2 \;=\; \frac{1}{I-j-1} \sum_{i=1}^{I-j} C_{i,j} \left(\frac{C_{i,j+1}}{C_{i,j}} - \hat{f}_j \right)^2, \qquad j = 1, \dots, J-2. \tag{3.4}$$

The estimator is unbiased for σ_j^2 under M1, M2, M3 — the proof is a direct computation that the column-j cumulative paid acts as the exposure weight

Monte Carlo Methods for Non-Life Actuaries delivers the full reserving-and-capital pipeline a Solvency II internal-model team needs but no single text on the actuary's shelf assembles. Glasserman speaks to derivatives books. Mack is the analytic core but is silent on cat, copula, and allocation. Klugman covers compound aggregates but stops short of the one-year horizon. The non-life carrier stitches them together from scratch every recalibration cycle.

Dr. Sigrid Norrgård — twenty-six years across If P&C reserving and Munich Re Stockholm capital modelling, sign-off actuary on roughly one hundred and forty year-end reserve reviews and twelve internal-model approval submissions — provides that stitch. Fifteen chapters cover the methods (bootstrap chain ladder, Mack analytic, Tweedie GLM, Bayesian MCMC, Panjer compound, GPD severity tail, t-copula aggregation, reinsurance layer pricing, ELT-driven cat, Merz–Wüthrich one-year, premium-risk cycle, Euler and Aumann–Shapley allocation) on real four-line calibrated portfolios. Every method ships with its R implementation — a working production stack you can run, fork, and walk into a year-end review with. The hundred-and-five-exercise appendix is the methodology defense in the actuary's voice: the questions a reviewer will ask, answered.

For the actuary whose next internal-model submission must read defensibly to the Finansinspektionen, BaFin, or the PRA — and who has stopped pretending the existing literature covers what the regulator actually asks.